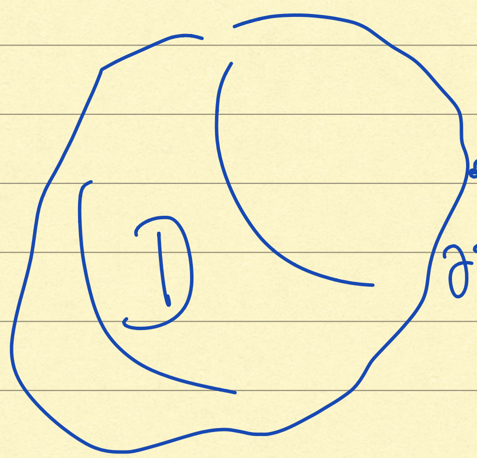
Teorema da divergência.

$D \subset \mathbb{R}^3$ aberto, limitado, regular

$F: \mathbb{R}^3 \to \mathbb{R}^3$, C^1 em D

$$\iiint_D \operatorname{div} F = \underbrace{\iint_{\partial D} F \cdot N_{ext}}_{\boxed{\text{Fluxo}}}$$



$\to N_{ext}$ (Normal exterior)

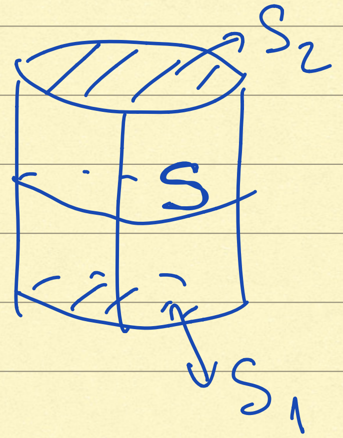
∂D = união de superfícies

$F = (P, Q, R)$

$$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$\partial D = S \cup S_1 \cup S_2 \cup \dots \cup S_k$

<u>Exemplo</u>: $D: \ x^2+y^2<1 \,;\ 0<z<1$



<u>Sólido</u>

$\partial D = S \cup S_1 \cup S_2$

$S: \ x^2+y^2=1 \,;\ 0<z<1$

$S_1: \ z=0 \,;\ x^2+y^2<1$

$S_2: \ z=1 \,;\ x^2+y^2<1$

<u>Problema</u>: calcula $\iint_S F \cdot N$

$$\iiint_D \operatorname{div} F = \iint_{\partial D} F \cdot N_{ext} \quad (\text{teorema})$$

$$= \boxed{\iint_S F \cdot N_{ext}} + \iint_{S_1} F \cdot N_{ext} + \cdots + \iint_{S_k} F \cdot N_{ext}$$

???

$$\iint_S F \cdot N_{ext} = \underbrace{\iiint_D \operatorname{div} F} - \underbrace{\iint_{S_1} F \cdot N_{ext}} - \cdots - \underbrace{\iint_{S_k} F \cdot N_{ext}}$$

Dados: F, S, N

de S construir D tal que

$\partial D = S \cup S_1 \cup S_2 \cup \cdots \cup S_k$

$S \subset \mathbb{R}^3$, superfície $\boxed{=}$ 1 equação

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

D aberto $\boxed{\text{limitado}}$ $\qquad$ $\geq$ ou $\leq$

Exemplo: $S: x^2+y^2=1;\ 0<z<1$

$F(x,y,z)=(1,2,3)$

$$\iint_S F \cdot N, \qquad N_2(0,1,\tfrac{1}{2}) \geq 0$$

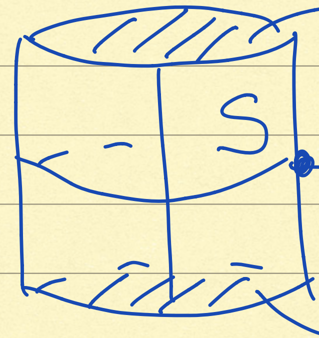
$\operatorname{div} F = 0$

$S: x^2+y^2 = 1;\ 0<z<1$

$\downarrow$

$D: x^2+y^2 \leq 1;\ 0<z<1$

$$\partial D = S \cup S_1 \cup S_2$$



$S_2 : z = 1$, $x^2+y^2<1$

$\boxed{N \equiv N_{ext}}$

$S_1 : z = 0$, $x^2+y^2<1$

$$\iiint_D \operatorname{div} F = \boxed{\iint_S F\cdot N_{ext}} + \iint_{S_1} F\cdot N_{ext} + \iint_{S_2} F\cdot N_{ext}$$

???

$\boxed{N \equiv N_{ext}}$

$\operatorname{div} F = 0$

$$\iint_S F\cdot N = -\left(\iint_{S_1} F\cdot N_{ext} + \iint_{S_2} F\cdot N_{ext}\right)$$

a calcular: $\iint_{S_1} F \cdot N_{ext}$, $\iint_{S_2} F \cdot N_{ext}$

S_1: $z=0$; $x^2+y^2 \leq 1$

$$\left.\begin{aligned} N_{ext} &= (0,0,-1) \\ F(x,y,0) &= (1,2,3) \end{aligned}\right\} F \cdot N_{ext} = -3$$

$$\iint_{S_1} F \cdot N_{ext} = -\iint_{S_1} 3 = -3\,vol_2(S_1)$$

$$= -3\pi$$

S_2: $z=1$; $x^2+y^2<1$

$$\left.\begin{aligned} N_{ext} &= (0,0,1) \\ F(x,y,1) &= (1,2,3) \end{aligned}\right\} F \cdot N_{ext} = 3$$

$$\iint_{S_2} F \cdot N_{ext} = 3\,vol_2(S_2) = 3\pi$$

$$\iint_S F\cdot N = -(-3\pi + 3\pi) = 0.$$

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Stokes: Rotacional de um campo $F: \mathbb{R}^3 \to \mathbb{R}^3$, C^1.

$F = (P, Q, R)$

$$\operatorname{rot} F = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

Mnemónica:

$$\operatorname{rot} F = \text{"} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} \text{"} \begin{array}{l} \\ \leftarrow \nabla \\ \leftarrow F \end{array}$$

$$\parallel$$

$$\nabla \times F$$

$$\mathrm{rot}\, F \equiv \nabla \times F$$

$$\| $$

$$F(x,y) = \boxed{(x, y)} \qquad \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} = 0$$

$$H(x,y) = \boxed{(-y, x)} \qquad \frac{\partial Q}{\partial x} = 1$$

$$\frac{\partial P}{\partial y} = -1$$